

قسم الهندسة الحيوية  
الطبية والمنظومات



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# Computer Vision 404 B Tutorial 3 25/03/20

It aims at fitting a curve of arbitrary shape to a set of object edge points

$$\sum_{i=1}^N (\alpha_i E_{cont} + \beta_i E_{curv} + \gamma_i E_{image})$$



# Internal Energy

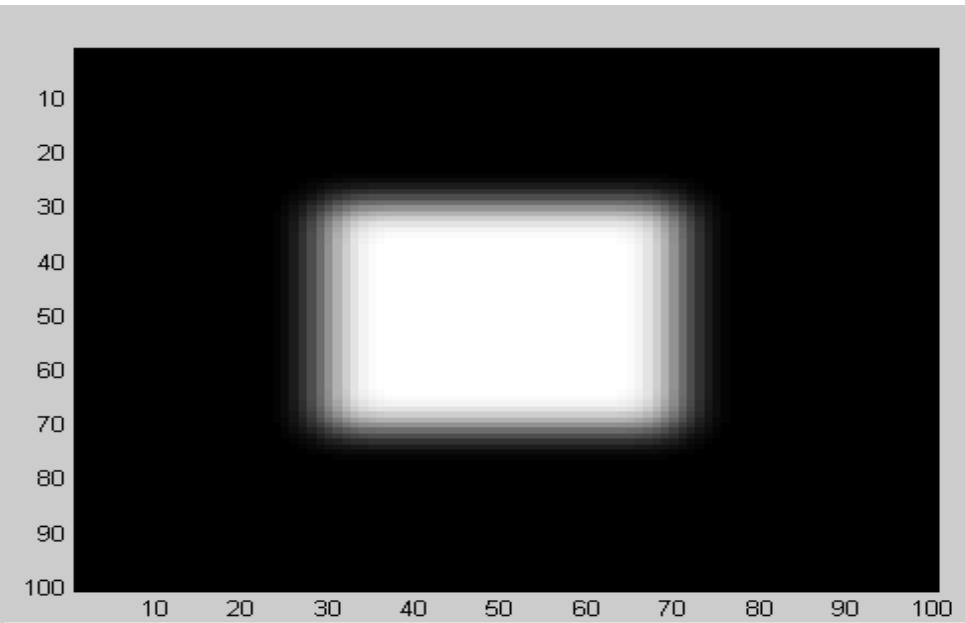
## Continuity Energy Term

$$E_{cont} = \|\mathbf{p}_i - \mathbf{p}_{i-1}\|^2 \quad \Rightarrow \quad E_{cont} = (\bar{d} - \|\mathbf{p}_i - \mathbf{p}_{i-1}\|)^2$$

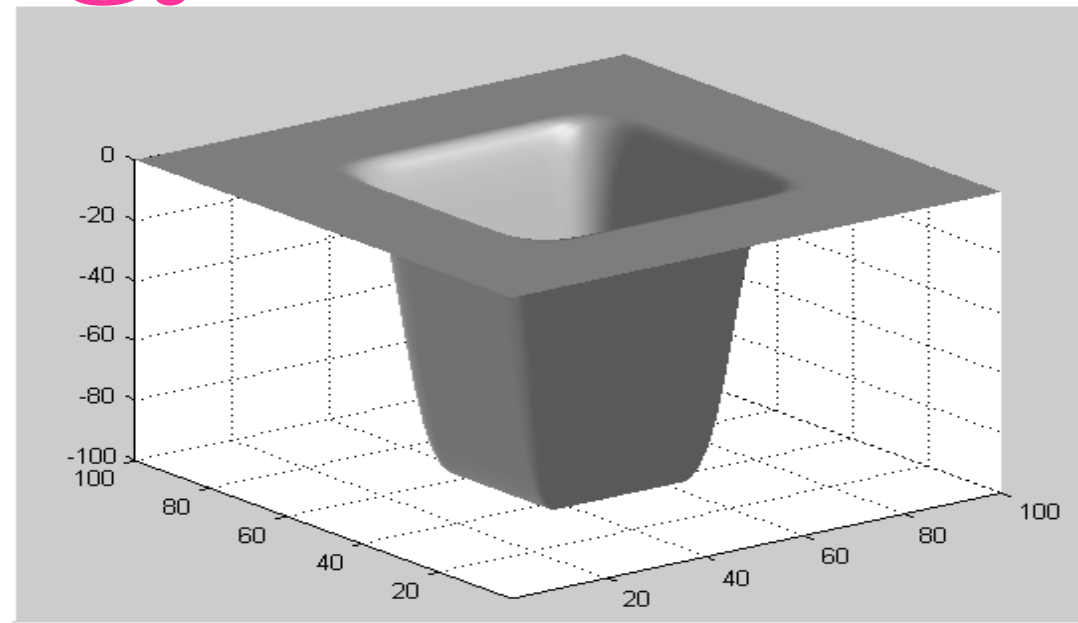
## Curvature Energy Term

$$E_{curv} = \|\mathbf{p}_{i-1} - 2\mathbf{p}_i + \mathbf{p}_{i+1}\|^2$$

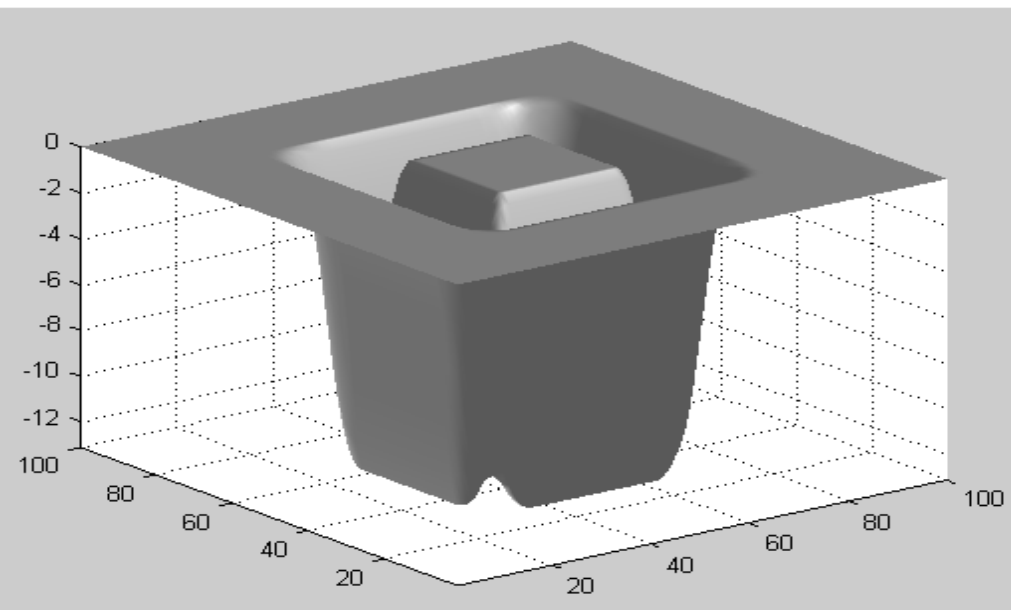
# External Energy: Image Energy



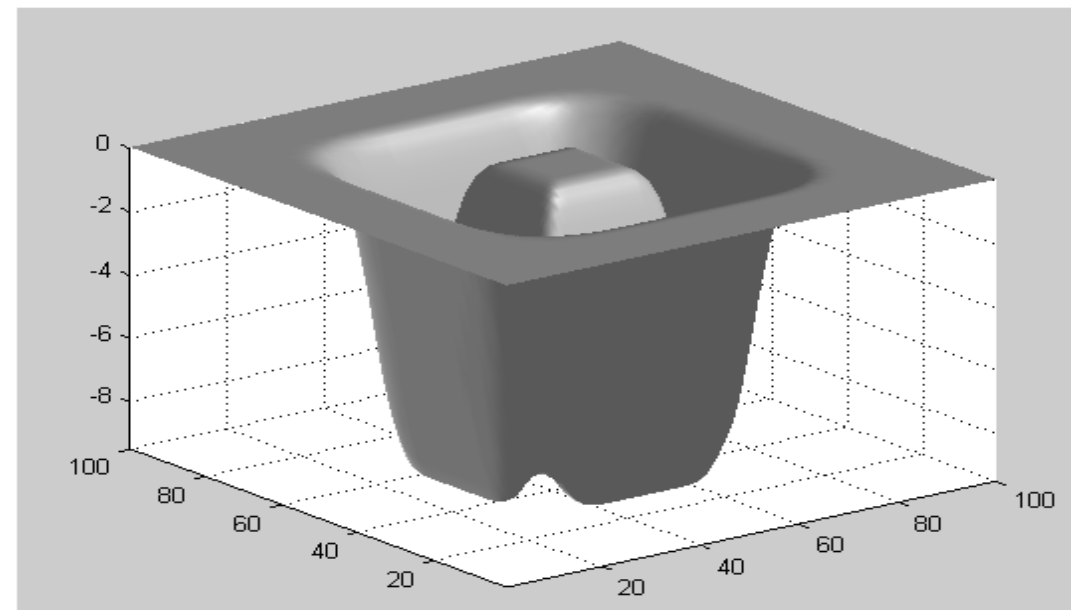
I



-I



25/3/2020  $|\nabla I(x, y)|$



$-\|\nabla[G_\sigma(x, y) * I(x, y)]\|$

# Weights: $\alpha$ , $\beta$ , and $\gamma$

E	$\alpha, \beta, \gamma$	Etot	Min P
1, 1, -1	1, 1, 1	1	
1, 1, -2	1, 1, 1	0	✓
1, 1, -1	1, 2, 1	2	
1, 1, -2	1, 2, 1	1	✓
1, 1, -1	1, 2, 1	2	✓
2, 1, -2	1, 2, 1	4	



**alpha=1 , Beta =1, Gamma =0.5**



**alpha=1, Beta=1, Gamma =1**



**Alpha=1, Beta =1, Gamma=2**

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**Alpha=1, Beta=1, Gamma =5**



Alpha=0.5, Beta=1, Gamma=1



Alpha=2, Beta=1, Gamma=1



Alpha=4, Beta=1, Gamma=1



Alpha=1, Beta=.5, gamma=1



Alpha=1. Beta=2. gamma=1



Alpha=1, Beta=4, Gamma=1





alpha = 5; beta = 2; gamma = 1.2

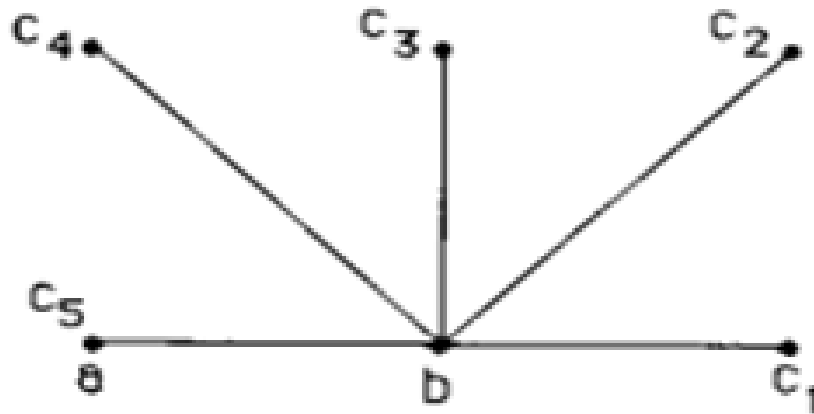


alpha = 5; beta = 1.5; gamma = 1.2

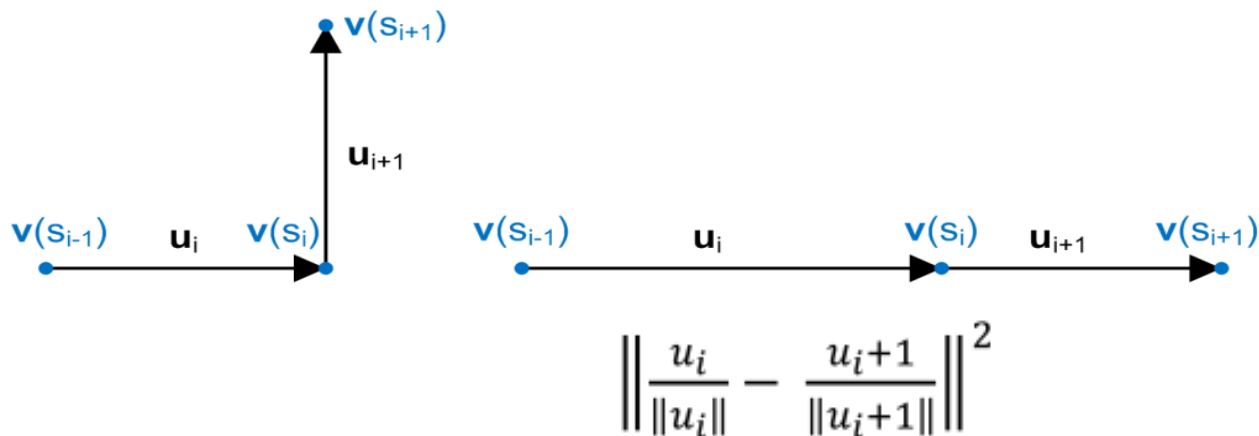


alpha = 5; beta = 1.5; gamma = 1

# Curvature: a-b-c



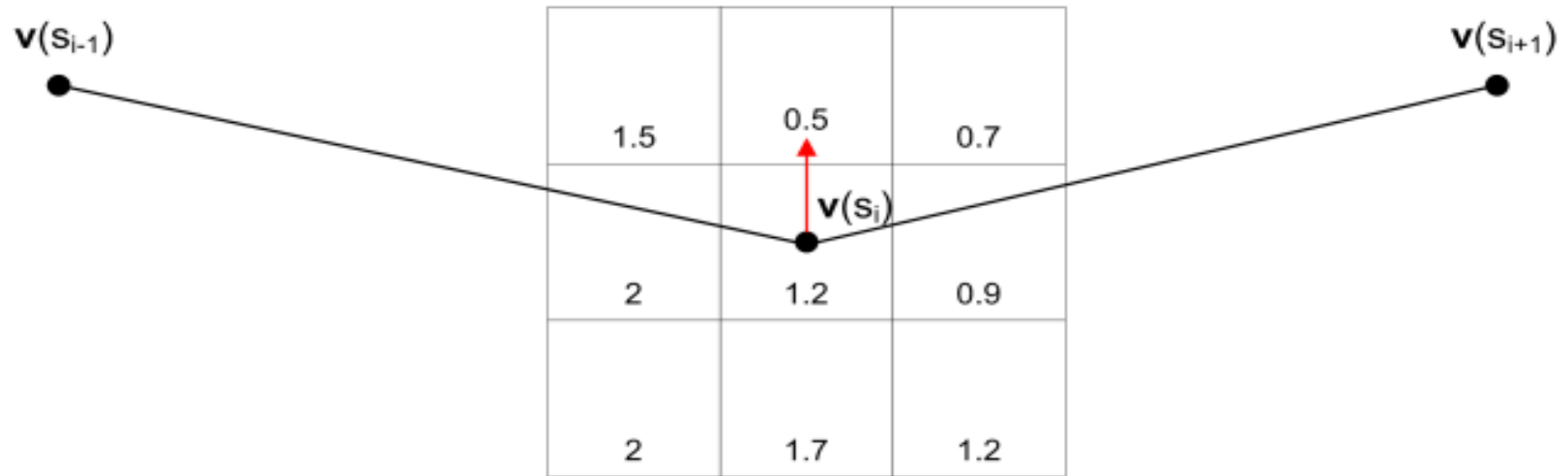
c	$\ \vec{u}_i - \vec{u}_{i+1}\ ^2$
1	0.0
2	1.0
3	2.0
4	5.0



where  $u_i = [x(s_i) - x(s_{i-1}), y(s_i) - y(s_{i-1})]$  and  $u_{i+1} = [x(s_{i+1}) - x(s_i), y(s_{i+1}) - y(s_i)]$

# Curvature: The two measures explained

$V_{i-1}$	$V_i$	$V_{i+1}$	$E_{curv}$	$U_i$	$U_{i+1}$	$C$	$\text{Cos}(\theta)$
1,1	3,1	3,3	8	2,0	0,2	2	0
1,1	6,1	6,3	29	5,0	0,2	2	0
1,1	3,1	5,1	0	2,0	2,0	0	-1
1,1	6,1	7,1	4	5,0	1,0	0	-1
1,1	7,1	3,5	116	6,0	-4,4	3.38	0.71
3,1	7,1	3,5	80	4,0	-4,4	3.38	0.71
1,1	7,1	2,2	125	6,0	-5,1	3.96	0.98
3,1	7,1	2,2	85	4,0	-5,1	3.96	0.98



Values shown in the cells correspond to  $E_{\text{total}}$

### Algorithm 1 A GREEDY SNAKE

Input: : determine ROI, parameters  $\alpha$ ,  $\beta$  and  $\gamma$

Output: Stop if only few points have moved

```
1 % n is the total number of snake control points
2 Index arithmetic for the snake control points is modulo n
3 Initialize the parameters  $\alpha$ ,  $\beta$  and  $\gamma$ 
4 Do % Main loop that moves the snake points to new locations
5   for i = 1 to n % The first and last point are the same in snake
6     Emin = infinity
7     for j = 1 to m % m is the neighborhood size
8       E(j) =  $\alpha$  Eela(j) +  $\beta$  Ecurv(j) +  $\gamma$  Eimg(j)
9       if E(j) < Emin then % Find location with min energy
10        Emin = E(j)
11        jmin = j
12        Move point v(i) to location jmin
13        if jmin is not the current location then ptsmoved ++
14 % The process below determines where to relax ?
15   for i = 1 to n % Calculate exact curvature
16     c(i) = || u(i) / || u(i) || - u(i+1) / || u(i+1) || || ^2
17   for i = 1 to n % Find where to relax ?
18     if (c(i) > c(i-1) and c(i) > c(i+1)
19       and c(i) > TH
20       and mag(v(i)) > TH-mag
21     then  $\beta$ (i) = 0 % Relax ? if all conditions true
22     while ptsmoved > TH-moved
23 % Stop if only few points have moved
```

Figure 3.4:Pseudo-code for the greedy snake algorithm[39].

# Snakes: Beta effect

The input is formed by an intensity image,  $I$ , which contains a closed contour of interest, and by a chain of image locations,  $\mathbf{p}_1, \dots, \mathbf{p}_N$ , defining the initial position and shape of the snake.

Let  $f$  be the minimum fraction of snake points that must move in each iteration before convergence, and  $U(\mathbf{p})$  a small neighborhood of point  $\mathbf{p}$ . In the beginning,  $\mathbf{p}_i = \bar{\mathbf{p}}_i$  and  $d = \bar{d}$  (used in  $E_{cont}$ ).

While a fraction greater than  $f$  of the snake points move in an iteration:

1. for each  $i = 1, \dots, N$ , find the location of  $U(\mathbf{p}_i)$  for which the functional  $\mathcal{E}$  is minimum, and move the snake point  $\mathbf{p}_i$  to that location;
2. for each  $i = 1, \dots, N$ , estimate the curvature  $k$  of the snake at  $\mathbf{p}_i$  as

$$k = |\mathbf{p}_{i-1} - 2\mathbf{p}_i + \mathbf{p}_{i+1}|,$$

and look for local maxima. Set  $\beta_j = 0$  for all  $\mathbf{p}_j$  at which the curvature has a local maximum and exceeds a user-defined minimum value;

3. update the value of the average distance,  $\bar{d}$ .

On output this algorithm returns a chain of points  $\mathbf{p}_i$  that represent a deformable contour.

- It is important to normalize the contribution of each term for correct implementation:
- 1) For  $E_{cont}$  and  $E_{curv}$ , it is sufficient to divide by the largest value in the neighborhood in which the point can move.
  - 2) Normalize  $E_{image}$  using this formula

$$new\_v = \frac{v - min_x}{max_x - min_x} \cdot (new\_max_x - new\_min_x) + new\_min_x$$

Which is reduced to

$$new\_v = \frac{v - min_x}{max_x - min_x}$$

When the array is normalized between 0 and 1

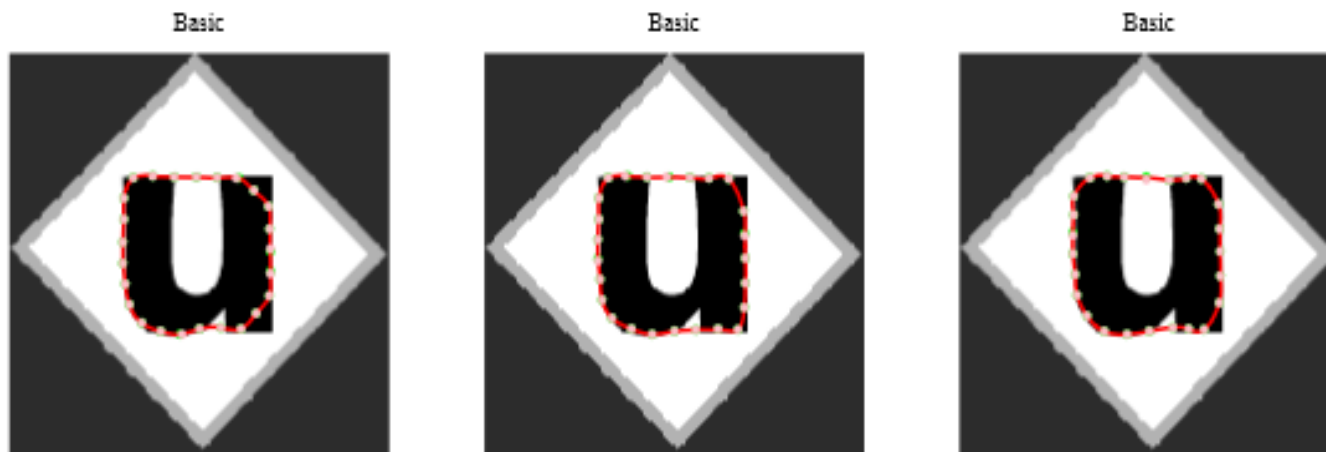


Figure 3.6: Result of the basic snake on a test image with  $\alpha = 1$  (left),  $\alpha = 0.5$  (middle) and  $\alpha = 0$  (right).

As can be seen in figure 3.6, by decreasing  $\alpha$  the distance between the contour points is less and less equal. When  $\alpha$  is equal to zero the contour points leave big gaps between each other.



Figure 3.7: Result of the basic snake on a test image with  $\beta = 1$  (left),  $\beta = 0.5$  (middle) and  $\beta = 0$  (right).

As can be seen in figure 3.7, by decreasing  $\beta$  the contour becomes less rigid and sharper angles can be formed between contour points. This can give dramatic results as in the right image. When  $\beta$  is too big though as in the left image, the contour points can't handle corners well. This can especially be seen by comparing the left image with the middle image, where the corners are taken much better.



# References

1) P. Tiilikainen, "A Comparative Study of Active Contour Snakes".  
Copenhagen University, Denmark, 2007

<https://www.scribd.com/document/324882550/nikolas-070901>

3) <https://www.cse.unr.edu/~lzhang/snake/snake.doc>

4) [https://pdfs.semanticscholar.org/562b/786ef6921c4770582905cc6ee1c3fa0fcee4.pdf?\\_ga=2.263374796.1921450585.1585126563-1685233747.1572623401](https://pdfs.semanticscholar.org/562b/786ef6921c4770582905cc6ee1c3fa0fcee4.pdf?_ga=2.263374796.1921450585.1585126563-1685233747.1572623401)