Problem 1

Calls to a telephone system follow a Poisson process with a mean of five calls per minute.

1. What is the name applied to the distribution and parameter values of the time until the 10th call?
2. What is the mean time until the 10th call?
3. What is the mean time between the 9th and 10th calls?
4. What is the probability that exactly four calls occur within 1 minute?
5. If 10 separate 1-minute intervals are chosen, what is the probability that all intervals contain more than two calls?

Solution

1. *Erlang distribution* (time is continuous RV), where $\lambda = 5$ and $r = 10$.
2. Using *Erlang* expectation operator, $E(X) = \frac{10}{5} = 2$ minutes
3. Since time between any two events follows an *Exponential distribution*, then $E(X) = \frac{1}{\lambda} = 0.2$ minutes
4. Time is given (fixed) and no. of calls is RV, which follows *Poisson distribution*, hence:

$$\begin{aligned} P(X = 4) &= \frac{(\lambda T)^x e^{-\lambda T}}{x!} \\ &= \frac{(5)^4 e^{-5}}{4!} \\ &= 0.1754674 \end{aligned}$$

5. First, probability of a more than two calls in a single interval follows a *Poisson distribution*, hence:

$$\begin{aligned} P(A) = P(X > 2) &= 1 - (P(2) + P(1) + P(0)) \\ &= 0.875348 \end{aligned}$$

   Second, Probability of ten intervals satisfy event A follows *Binomial distribution*, hence:

$$P(X = 10) = \text{binom}(10; n = 10, p = 0.875348) = 0.2641237$$