Problem 1

A civil engineer is analyzing the compressive strength of concrete. Compressive strength is normally distributed with $\sigma^2 = 1000(\text{psi})^2$. A random sample of 12 specimens has a mean compressive strength of $\bar{x} = 3250psi$.

a. Construct a 95% two-sided confidence interval on mean compressive strength.

b. Construct a 99% two-sided confidence interval on mean compressive strength. Compare the width of this confidence interval with the width of the one found in part (a).

Solution

1. Population is normal with known variance, then we use z-distribution regardless the sample size:

$$\bar{x} - z_{0.025}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{0.025}\frac{\sigma}{\sqrt{n}}$$

$$3250 - 1.96\frac{\sqrt{1000}}{\sqrt{12}} \leq \mu \leq 3250 + 1.96\frac{\sqrt{1000}}{\sqrt{12}}$$
$$3232.108 \leq \mu \leq 3267.892$$

2. Population is normal with known variance, then we use z-distribution regardless the sample size:

$$\bar{x} - z_{0.005}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{0.005}\frac{\sigma}{\sqrt{n}}$$

$$3250 - 2.756\frac{\sqrt{1000}}{\sqrt{12}} \leq \mu \leq 3250 + 2.756\frac{\sqrt{1000}}{\sqrt{12}}$$
$$3226.486 \leq \mu \leq 3273.514$$

The 99%-CI is larger than 95%-CI.

Problem 2

A civil engineer is analyzing the compressive strength of concrete. Compressive strength is normally distributed with $\sigma^2 = 1000(\text{psi})^2$. A random sample of 18 specimens has a mean compressive strength of $\bar{x} = 3250psi$.

a. Construct a 95% two-sided confidence interval on mean compressive strength.

b. Construct a 90% two-sided confidence interval on mean compressive strength. Compare the width of this confidence interval with the width of the one found in part (a).

Solution

1. Population is normal with known variance, then we use z-distribution regardless the sample size:

$$\bar{x} - z_{0.025}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{0.025}\frac{\sigma}{\sqrt{n}}$$

$$3250 - 1.96\frac{\sqrt{1000}}{\sqrt{18}} \leq \mu \leq 3250 + 1.96\frac{\sqrt{1000}}{\sqrt{18}}$$
$$3235.391 \leq \mu \leq 3264.609$$

2. Population is normal with known variance, then we use z-distribution regardless the sample size:

$$\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{0.05}\frac{\sigma}{\sqrt{n}}$$

$$3250 - 1.64\frac{\sqrt{1000}}{\sqrt{18}} \leq \mu \leq 3250 + 1.64\frac{\sqrt{1000}}{\sqrt{18}}$$
$$3237.74 \leq \mu \leq 3262.26$$

The 95%-CI is larger than 90%-CI.

Problem 3

The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. A sample of 10 tubes results in $\bar{x} = 317.2$ and $s = 15.7$.

1. Find (in microamps) a 99% two-sided confidence interval on mean current required. State any necessary assumptions about the underlying distribution of the data.

Solution

$$\begin{aligned} \bar{x} - t_{0.005,9}\frac{s}{\sqrt{n}} \leq &\mu \leq \bar{x} + t_{0.005,9}\frac{s}{\sqrt{n}} \\ 317.2 - 3.25\frac{15.7}{\sqrt{10}} \leq &\mu \leq 317.2 + 3.25\frac{15.7}{\sqrt{10}} \\ 301.06 \leq &\mu \leq 333.33 \end{aligned}$$

Since we have small sample with unknown population variance, the population has to be assumed normal in order to have our estimate valid.

Problem 4

The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. A sample of 10 tubes results in $\bar{x} = 317.2$ and $s = 15.7$.

1. Find (in microamps) a 95% one-sided lower confidence interval on mean current required. State any necessary assumptions about the underlying distribution of the data.

Solution

$$\begin{aligned} &\mu \leq \bar{x} + t_{0.05,9}\frac{s}{\sqrt{n}} \\ &\mu \leq 317.2 + 1.833\frac{15.7}{\sqrt{10}} \\ &\mu \leq 326.3 \end{aligned}$$

Since we have small sample with unknown population variance, the population has to be assumed normal in order to have our estimate valid.