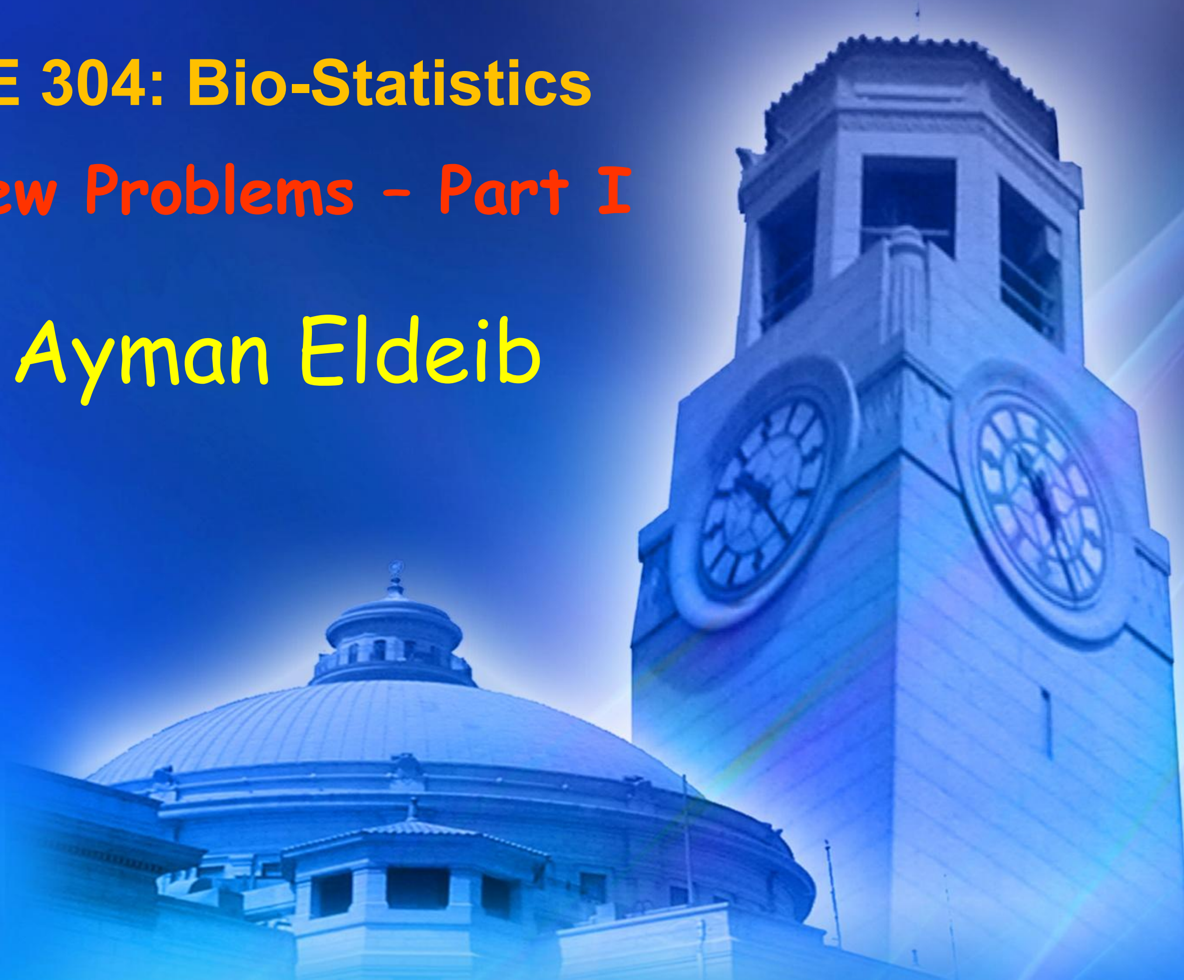
Systems & Biomedical Engineering Department

SBE 304: Bio-Statistics

Review Problems – Part I



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SBE 304: Review Problems – Part I

Experience has shown that 30% of all persons afflicted by a certain illness recover. A drug company has developed a new medication. Ten people with the illness were selected at random and received the medication; nine recovered shortly thereafter. Suppose that the medication was absolutely worthless. What is the probability that at least nine of ten receiving the medication will recover?

Let X denote the number of people who recover

If the medication is worthless, the probability that a single ill person will recover = 0.3

Number of trials = n = 10

X is a binomial random variable

$P(X \geq 9) = p(9) + p(10) = 0.000144$

SBE 304: Review Problems – Part I

Suppose that the number of patients that enter a hospital in an hour is a Poisson random variable, and suppose that

$$P(X = 0) = 0.03$$

Determine the mean and variance of *X*.

$P(X = 0) = e^{-\lambda} = 0.03$ Therefore, $\lambda = -\ln(0.03) = 3.51$

Consequently, $E(X) = V(X) = 3.51$

SBE 304: Review Problems – Part I

If the range of X is the set {0, 1, 2, 3, 4} and $P(X = x) = 0.2$ determine the expected value and variance of the random variable.

$$\mu = E(X) = 0(0.2) + 1(0.2) + 2(0.2) + 3(0.2) + 4(0.2) = 2$$

$$\begin{aligned} V(X) &= E(X^2) - \mu^2 \\ &= 0(0.2) + 1(0.2) + 4(0.2) + 9(0.2) + 16(0.2) - 4 = 2 \end{aligned}$$

Uniform?

What if $P(X = x) = 0.4$? Is it possible in the above example?

SBE 304: Review Problems – Part I

Let the random variable X have a discrete uniform distribution on the integers $1 \le x \le 3$. Determine the mean and variance of X.

$E(X) = (3+1)/2 = 2$

$E(X) = 1\,(1/3) + 2\,(1/3) + 3\,(1/3) = 2$

$V(X) = 2/3$

SBE 304: Review Problems – Part I

The random variable X has a binomial distribution with n =10 and p = 0.5. Determine the following probabilities:

$P(X=5)$ $P(X\le 2)$ $P(3 \le X<5)$

$$P(X=5) = \binom{10}{5} * 0.5^{5} * 0.5^{5} = 0.2461$$

$$P(X\le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$P(3 \le X<5) = P(X=3) + P(X=4)$$

SBE 304: Review Problems – Part I

Thickness measurements of a coating process are made to the nearest hundredth of a millimeter. The thickness measurements are uniformly distributed with values 0.15, 0.16, 0.17, 0.18, and 0.19. Determine the mean and variance of the coating thickness for this process.

$X=(1/100)Y, \qquad Y = 15, 16, 17, 18, 19.$

$$E(aY) = aE(Y)$$

$$E(X) = aE(Y) \; =(1/100)\, E(Y)$$

$$V(aY+ b) = a^2V(Y)$$

SBE 304: Review Problems – Part I

A hospital has eight computers that are uses for the Hospital Information System (HIS). The probability of a computer failing in a day is 0.005, and the computers fail independently. Computers are repaired in the evening and each day is an independent trial.

(a) What is the probability that all eight computers fail in a day?

(b) What is the mean number of days until a specific computer fails?

(c) What is the mean number of days until all eight computers fail in the same day?

$$P(X = 8) = 0.005^8 = 3.91\text{x}10^{-19}$$

$$E(X) = 1/p = 1/0.005 = 200 \text{ days}$$

Let Y denote the number of days until all eight computers fail in the same day

$$E(Y) = 1/p = 1/(3.91\text{x}10^{-19}) = 2.56 * 10^{18} \text{ days}$$

SBE 304: Review Problems – Part I

A company employs 800 men under the age of 55. Suppose that 30% carry a marker on the male chromosome that indicates an increased risk for high blood pressure.

(a) If 10 men in the company are tested for the marker in this chromosome, what is the probability that exactly 1 man has the marker?

(b) If 10 men in the company are tested for the marker in this chromosome, what is the probability that more than 1 has the marker?

Let X denotes the number of men who carry the marker on the male chromosome for an increased risk for high blood pressure.

$N=800$, $K=240$ $n=10$

Then X is a hypergeometric random variable

SBE 304: Review Problems – Part I

Let X denotes the number of men who carry the marker on the male chromosome for an increased risk for high blood pressure.

N=800, K=240 n=10

Then X is a hypergeometric random variable

$$p(X=1)=\frac{\binom{K}{1}\binom{N-K}{n-1}}{\binom{N}{n}} = 0.1201$$

$$P(X>1) = 1 - P(X \le 1) = 1 - [P(X=0) + P(X=1)] = 0.8523$$

SBE 304: Review Problems – Part I

Assume that each of your calls to a hospital in an emergency situation has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.
(a) What is the probability that your first call that connects is your tenth call?
(b) What is the probability that it requires more than five calls for you to connect?
(c) What is the mean number of calls needed to connect?

X is a geometric random variable with p =0.02

$P(X = 10) = (1 - 0.02)^9 * 0.02 = 0.98^9 * 0.02 = 0.0167$

$P(X > 5) = 1 - P(X \leq 5)$

$= 1 - [P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)]$

$E(X) = 1/0.02 = 50$

SBE 304: Review Problems – Part I

The weekly amount of money spent on maintenance and repairs by a doctor office was observed over a long period of time, to be approximately normally distributed with mean 400 L.E. and standard deviation 20 L.E.

a. If 450 L.E. is budgeted for next week, what is the probability that the actual costs will exceed the budgeted amount?

$$P\left(Y > 450\right) = P(Z > \frac{450-400}{20}) = P\left(Z > 2.5\right) = 0.0062$$

b. How much should be budgeted for weekly repairs and maintenance to provide that the probability the budgeted amount will be exceeded in a given week is only 0.10?

$$P\left(Y > y_o\right) = 0.1$$

$$P\left(Z > \frac{y_o - 400}{20}\right) = 0.1 \qquad \therefore z = \frac{y_o - 400}{20} = 1.28$$

$$\therefore y_o = 400 + 20\left(1.28\right) = 425.6\ L.E.$$

SBE 304: Review Problems – Part I

Consider the following two samples:

Sample 1:	10,	9,	8,	7,	8,	6,	10,	6
Sample 2:	10,	6,	10,	6,	8,	10,	8,	6

a. Calculate the sample range for both samples. Would you conclude that both samples exhibit the same variability? Explain.

Sample 1 - Range = 4 **Sample 2 - Range = 4**

Yes, the two appear to exhibit the same variability

a. Calculate the sample standard deviations for both samples. Do these quantities indicate that both samples have the same variability? Explain.

Write a short statement contrasting the sample range versus the sample standard deviation as a measure of variability.

Sample 1 -s = 1.604 **Sample 2 - s = 1.852**

No, sample 2 has a larger standard deviation.

The sample range is a relatively crude measure of the sample variability as compared to the sample standard deviation since the standard deviation uses the information from every data point in the sample whereas the range uses the information contained in only two data points - the minimum and maximum.

SBE 304: Review Problems – Part I

In an automated filling operation of a certain drug, the probability of an incorrect fill when the process is operated at a low speed is 0.001. When the process is operated at a high speed, the probability of an incorrect fill is 0.01. Assume that 30% of the containers are filled when the process is operated at a high speed and the remainders are filled when the process is operated at a low speed.

a. What is the probability of an incorrectly filled container?

0.01 * .3 + 0.001 * .7 = 0.0037

b. If an incorrectly filled container is found, what is the probability that it was filled during the high-speed operation?

0.01 * 0.3 / 0.0037 = 0.811

SBE 304: Review Problems – Part I

An electronic scale in this automated filling operation stops the manufacturing line after three incorrect filling are detected. Suppose that each fill is independent.

c. What is the mean number of fills before the line is stopped?

Let X denote the number of fills needed to detect three underweight packages. Then X is a negative binomial random variable with $p = 0.0037$ and $r = 3$.

$E(X) = 3/0.0037 = 810.8$; i.e. 811

d. What is the standard deviation of the number of fills before the line is stopped?

$V(X) = [3(0.9963)/0.0037^2] = 218327$. Therefore, $\sigma_x = 467.26$

SBE 304: Review Problems – Part I

The fraction of defective patient monitors produced in a new production line is being studied.

A random sample of 300 monitors is tested, revealing 13 defectives. Find a 95% two-sided CI on the fraction of defective monitors produced by this new production line.

$$\boldsymbol{Z = (X - np)/\mathrm{sqrt}(np(1 - p)) = P - p \,/\, \mathrm{sqrt}(p(1 - p)/n)}$$

A 95% two-sided confidence interval for p can be computed as follows:
$p = 13/300 = 0.04$

$$\mathbf{0.04 - 1.96{*}sqrt(0.04(0.96)/300) \le p \le 0.04 + 1.96{*}sqrt(0.04(0.96)/300)}$$

$$\mathbf{0.04 - 0.02 \le p \le 0.04 + 0.02}$$

$$\mathbf{0.02 \le p \le 0.06}$$

SBE 304: Review Problems – Part I

The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.4 seconds and a standard deviation of 0.05 seconds.

a. What is the probability that a reaction requires more than 0.5 seconds?
b. What is the probability that a reaction requires between 0.4 and 0.5 seconds?
c. What is the reaction time that is exceeded 90% of the time?

a) **$P(Z > (.5 - .4)/0.05) = P(Z > 2) = P(Z < -2) = 0.0228$**

b) **$P(X < 0.5) – P(X < 0.4) = P(Z < 2) – 0.5$**
$= 1 – P(Z >= 2) – 0.5 = 1 – 0.0228 - 0.5 = 0.4772$

c) **$Z = -1.28 = (X – 0.4)/0.05$**
$X = -1.28 * 0.05 + 0.4 = 0.336$

Quiz

Thank You